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Does temperature contain a stochastic trend: linking statistical results to physical mechanisms

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Abstract By construction, the time series for radiative forcing that are used to run the 20c3m experiments, which are implemented by climate models, impart non-stationary movements (either stochastic or deterministic) to the simulated time series for global surface temperature. Here, we determine whether stochastic or deterministic trends are present in the simulated time series for global surface temperature by examining the time series for radiative forcing. Statistical tests indicate that the forcings contain a stochastic trend against the alternative hypothesis that the series are trend stationary with a one-time structural change. This result is consistent with the economic processes that impart a stochastic trend to anthropogenic emissions and the physical processes that integrate emissions in the atmosphere. Furthermore, the stochastic trend in the aggregate measure of radiative forcing also is present in the simulated time series for global surface temperature, which is consistent with the relation between these two variables that is represented by a zero dimensional energy balance model. Finally, we propose that internal weather variability imposed on the stochastic trend in radiative forcings is responsible for statistical results, which gives the impression that global surface temperature is trend stationary with a one-time structural change. We conclude that using the ideas of stochastic trends, cointegration, and error correction can generate reliable conclusions regarding the causes of changes in global surface temperature during the instrumental temperature record.

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1 Introduction

Univariate analyses of global surface temperature and the factors that generate changes during the instrumental record (Bloomfield and Nychka, 1992; Woodward and Gray, 1993; 1995) generate a long-running debate about their time series properties. This debate can be summarized as follows. One set of analysts postulate that the time series for the instrumental temperature record contains a stochastic trend and this trend is imparted by (and therefore cointegrates with) stochastic trends in the time series for solar insolation and the atmospheric concentrations of radiatively active gases, such as carbon dioxide, methane, CFCs, nitrous oxide, and sulfur (Stern and Kaufmann, 2000; Kaufmann and Stern, 2002; Kaufmann et al., 2006a; Mills, 2009; Kaufmann et al., 2011). A simple data generating process for the stochastic trends in radiative forcing and temperature is given by:

$$Y_t = \theta Y_{t-1} + \eta_t \tag{1}$$

in which Y is either temperature (Temp) or radiative forcing (F) at time t, θ is an autocorrelation coefficient, and η is a random error term whose mean may be non-zero. The time series Y contains a stochastic trend when $\theta=1$. If the stochastic trend in radiative forcing imparts a stochastic trend to temperature, their long-run relation is given by:

$$Temp_t = \alpha_1 + \beta F_t + \mu_t \tag{2}$$

in which Temp is temperature, F is radiative forcing, and μ is a stationary error term. The stationary nature of μ is critical—it implies that the stochastic trends in temperature and radiative forcing can be eliminated via the linear combination given by α_1 and β. This is known as cointegration. Cointegration is possible only if temperature and radiative forcing share the same stochastic trend. From this perspective, stochastic trends can be viewed as 'fingerprints' that are 'matched' with a finding of cointegration. The dynamics of the long-run cointegrating relation is examined using an error correction model:

$$\Delta Temp_t = \alpha_2 + \rho\mu_{t-1} + \sum_{i=1}^{s}\phi_i \Delta Temp_{t-i} + \sum_{i=1}^{s}\psi_i \Delta F_{t-i} + \varepsilon_t \tag{3}$$

in which ρ is the error correction coefficient, which quantifies the rate at which temperature adjusts to the disequilibrium μ in the long-run relation between radiative forcing and temperature that is given by Eq. (2), Δ is the first difference operator (e.g. $F_t - F_{t-1}$), and ε is the regression residual ($\varepsilon_t \sim Niid$(o, Ω).).

Conversely, other analysts argue that the time series for global surface temperature can be described as a trend-stationary process, with or without a one-time structural change (Gay et al., 2009; Estrada et al., 2010). According to this perspective, the data generating process for temperature (and radiative forcing) is given by:

$$Y_t = \alpha_3 + \gamma t + \lambda(t - T_b)(t > T_b) + e_t \tag{4}$$

in which t measures the passage of time, T_b is the date of a one-time structural change at which the slope of the time trend is altered by λ, and e_t is a stationary error term. According to this model, temperature and/or radiative forcing changes on average by the same mean quantity (γ) year after year, until there is a structural change, after which temperature and/or radiative forcing changes by a different average rate ($\gamma + \lambda$) year after year.

A third statistical approach is to model temperature as fractionally integrated or exhibiting long-run dependence, see for example Bloomfield and Nychka (1992) and Rea et al.(2011). Mann (2011) argues that, in finite samples, temperature data generated by a zero-dimensional energy balance model, with historical forcings, is capable of generating temperature data that appear to exhibit long-run dependence, but by construction do not. We adopt and extend Mann's (2011) reasoning below to show that this model implies cointegration between temperature and radiative forcing.

Resolving the differences between these two statistical approaches to modeling the relation between temperature and radiative forcing is important. From a technical perspective, using statistical techniques consistent with the underlying data generating process is critical to an accurate interpretation of results. Most importantly, the presence of stochastic trends invalidates the blind application of standard statistical techniques such as ordinary least squares (OLS) because they may generate spurious regression results (Kaufmann et al. 2006a). Equally important, the interpretation of statistical results depends on the model's ability to quantify the physical and economic mechanisms that generate anthropogenic emissions of radiatively active gases, the physical and chemical processes that determine their atmospheric lifetime, and the physical processes by which changes in radiative forcing generate changes in global surface temperature. Finally, statistical models that link temperature to radiative forcing, as opposed to a time trend, allow statistical models to generate forecasts based on explicit scenarios for the economic and physical determinants of emissions and concentrations of radiatively active gases.

Here, we explore the economic and physical underpinnings of competing statistical approaches by analyzing the time series properties of inputs to, and outputs from coupled atmosphere ocean general circulation models (AOGCM's) as suggested by Estrada et al. (2011). Specifically, we test whether the time series for radiative forcing used to simulate the 20c3m experiments and the resultant time series for global surface temperature contain a stochastic trend (Eq. 1) or are trend stationary with a one-time structural change (TSOTSC – Eq. 4).

Extending the analysis to experiments run by AOGCM's is important because it helps identify the data generating process for global surface temperature. Unlike analyses of real-world data, which focus on attribution (i.e. what causes the observed change in global surface temperature), the experimental design explicitly identifies the forces that impart non-stationary movements (either stochastic or deterministic) to the time series for global surface temperature that are simulated by climate models—the exogenous forcings that are used to simulate the 20c3m experiments. For each experiment, the AOGCM is subject to a constant level of forcing and 'spun-up' to equilibrium. At equilibrium, temperature and the other climate variables are stationary. The 20c3m experiments are run by subjecting the equilibrium state to changes in the radiative forcing associated with solar insolation, greenhouse gases, anthropogenic aerosols, and volcanic eruptions that are consistent with the historical record. These forcings are thus the source of any stochastic or deterministic trends in the experimental temperature data.

The unambiguous link between exogenous model inputs (radiative forcing) and endogenous model outputs (global surface temperature) suggests that the time series properties of the exogenous forcing variables that are used to run the experiment contain critical information about the nature of the resultant time series for global surface temperature. If the input time series for radiative forcing contain a stochastic trend, one would expect this trend to be present in the time series for global surface temperature. Conversely, if the input time series for radiative forcing are trend stationary with a one (or more) time structural change (TSOTSC), it is likely that the time series for global surface temperature will be TSOTSC.

Here, we analyze model inputs and outputs to expand the debate about how to estimate the relation between temperature and radiative forcing using statistical techniques. We start with a technical focus—can we chose between modeling techniques based on the time series properties of the data for radiative forcing and their relation to the resultant time series for global surface temperature? Statistical tests generally fail to reject the null hypothesis that the forcings contain a stochastic trend against the alternative hypothesis that the series are stationary or trend stationary with one or more structural changes. Furthermore, the same stochastic trend in radiative forcing also is present in the simulated time series for global surface temperature. These results are consistent with the use of cointegration/error correction models (Eqs. 2–3) to analyze the relation between radiative forcing and global temperature.

Next, we go beyond statistical issues to consider the degree to which the models embodied by Eqs. (1–3) or Eq. (4) are consistent with the economic, chemical, and physical processes that generate emissions, accumulate emissions into concentrations, and translate concentrations (and radiative forcing) into temperature. This discussion indicates that the presence of a stochastic trend is consistent with the economic processes that generate emissions, the atmospheric lifetimes of various gases, and that the cointegration/error correction model is consistent with the physical relation between radiative forcing and temperature, as represented by a zero-dimensional energy balance model. Conversely, there is no obvious link between a deterministic time trend and either emissions, concentrations, or temperature. Instead, statistical results that are consistent with the hypothesis that temperature is trend stationary with a one-time structural change may be generated by 'weather noise,' which is associated with the stochastic heat flux forcing of the ocean surface. Combining the technical results of statistical tests with this analysis of the economic, chemical, and physical mechanisms, we conclude that global surface temperature contains a stochastic trend that is imparted by radiative forcing.

2 Methodology

Data We use two sets of annual data: (1) the forcing data used to simulate the 20c3m experiments, and (2) the time series of temperature that are simulated by the models. The sources of the forcing that are used to simulate the 20c3m experiments are described by (http://www.mri-jma.go.jp/Dep/cl/cl4/IPCC-AR4/simulations2.html). Simulations are named by the model that generates the data. Tracing back to the original sources described therein, data on greenhouse gas concentrations between 1850 and 2000 are obtained from Hansen and Sato (2001). Data for solar insolation (watts per square meter) between 1850 and 2000 are obtained from Lean (2004). Finally, a time series for the radiative forcing due to volcanic eruptions is obtained from Sato et al. (1993). Concentrations are converted to radiative forcing using the formulae described by Stern and Kaufmann (2000).

Consistent with the physics of the climate models, these data are summed to generate an aggregate time series for radiative forcing. We test four series; greenhouse gases (CO_2, CH_4, N_2O, CFC11, CFC12), anthropogenic sulfur emissions (direct effects only[1]), solar insolation, and volcanic forcings. In addition, we create an aggregate for all forcings with a stochastic trend (greenhouse gases, sulfur emissions, and solar insolation). The definition for this aggregate is similar to the independent variable specified by previous statistical analyses of the relation between radiative forcing and global surface temperature (Stern and Kaufmann, 2000; Kaufmann and Stern, 2002; Kaufmann et al., 2006a; Mills, 2009; Kaufmann et al., 2011). Although all series are measured in watts per meter square, we do not include volcanic forcing in the aggregate because previous analyses indicate that this forcing is stationary and cannot be used to estimate temperature sensitivity (Lindzen and Giannitsis, 1998; Harvey and Kaufmann, 2002).

Statistical methods Each time series for radiative forcing is tested for the presence of a stochastic trend against an alternative hypothesis that it is stationary or trend stationary with a one-time break. The augmented Dickey Fuller (ADF) statistic (Dickey and Fuller, 1979) is used to test the null hypothesis that the time series has a stochastic trend against the alternative

[1] The 20c3m experiments specify the direct effects of sulfur emissions only.

hypothesis that the time series is stationary, possibly around a deterministic trend. As such, failure to reject the null hypothesis is consistent with the notion that the time series for radiative forcing contains a stochastic trend.

The hypothesis that the time series for radiative forcing can be modeled as trend stationary with one or more structural changes is assessed with test procedures developed by Perron (1997) and Lumsdaine and Papell (1997). In both tests, the null hypothesis is that the time series has a unit root, or in other words, a stochastic trend. In the Perron test, the alternative hypothesis is that the time series is trend stationary with a single break point (we allow a change in the slope of the trend, but both segments of the trend function are joined at the break point — the "additive outlier model" in Perron's terminology). Visual inspection and previous studies (Gay et al. (2009), Estrada et al. (2010)) suggests that a single break point is possible, but the test procedure does not require one to specify the break point a priori. The Lumsdaine and Papell test is more general in that it allows two break points in the trend function, again the break points are determined endogenously by the procedure. The rejection of the unit root null hypothesis of these tests provides evidence against a stochastic trend in the time series and it can be interpreted as supportive evidence in favor of the alternative hypothesis that the time series is trend stationary with potential breaks in the trend function. This conclusion is however uncertain, because the rejection of the null does not necessarily imply rejection of a unit root per se, but may imply rejection of a unit root without a break. As our test results provide no convincing evidence against the null of a unit root, we do not assess this question thoroughly in the present paper.

To test whether the stochastic trend in the time series for radiative forcing is present in the time series for global surface temperature that is simulated by the climate models, we test whether temperature and radiative forcing cointegrate. To do so, ordinary least squares is used to estimate Eq. (2), in which F is the aggregate of radiative forcings that contain a stochastic trend (greenhouse gases, anthropogenic sulfur emissions, and solar insolation) and temperature is the time series simulated by the AOGCM. The regression residual from Eq. 2 is tested for the presence of a stochastic trend using the ADF statistic. Values of the ADF statistic that reject the null hypothesis indicate that the residual is stationary, which implies that *Temp* and *F* share the same stochastic trend (i.e. temperature and radiative forcing cointegrate).

3 Results

The analysis of the time series for radiative forcing generally are consistent with the hypothesis that the variables that drive the 20c3m experiments contain a stochastic trend and are inconsistent with the hypothesis that these forcings can be modeled as stationary or trend stationary with a one-time structural change (Fig. 1). The ADF statistic fails to reject the null hypothesis that the time series contain a stochastic trend for all time series except the radiative forcing due to volcanic eruptions (Table 1). These results are consistent with those generated by previous analyses (e.g. Stern and Kaufmann, 2000; Kaufmann et al., 2006a).

Furthermore, these results are consistent with the results generated by the test statistic developed by Perron (1997). This test statistic generally fails to reject the null hypothesis that the forcing variables contain a stochastic trend against the alternative hypothesis that they are trend stationary with a one-time structural change (Table 1). The only exception is the forcing due to volcanic eruptions. This variable is most likely stationary—there is no geophysical reason to believe that volcanic eruptions increase or decrease deterministically over the sample period with a one-time structural break. The Lumsdaine and Papell (1997) test statistic provides some evidence against a unit root in sulfur emissions, solar activity and volcanic forcings when the alternative hypothesis is trend stationarity with two breaks, but

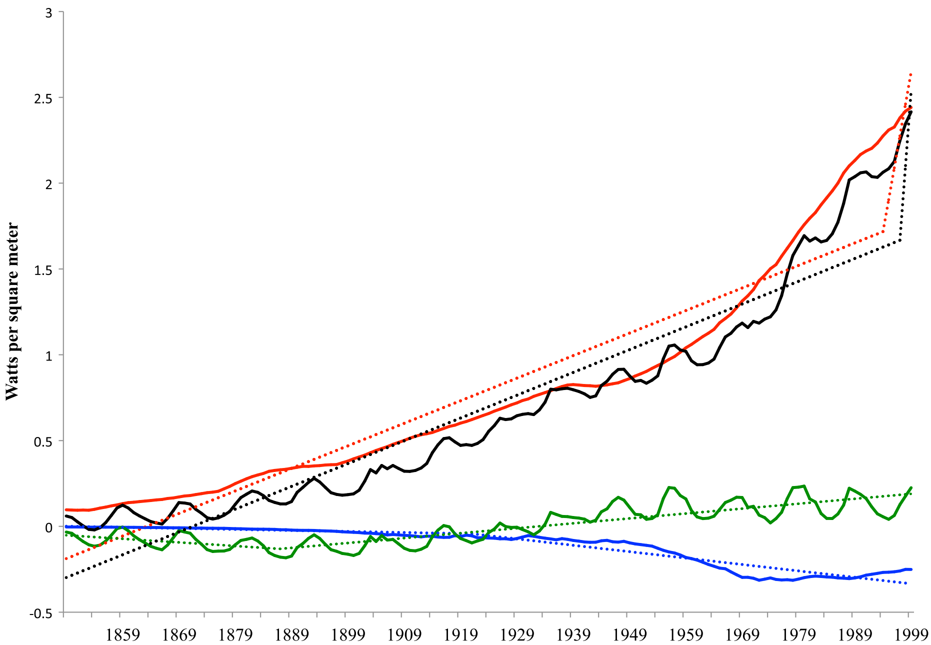

Fig. 1 The radiative forcings of greenhouse gases (*red line*), solar insolation (*green line*), anthropogenic sulfur emissions (*blue line*) and their sum (*black line*). *Dotted line* of same color represents the "best fit" trend stationary with a one-time break model

the test fails to reject the null hypothesis of a stochastic trend in the aggregate of radiative forcings. Together, these results indicate that the forcings used to simulate the 20c3m models contain a stochastic trend and cannot be modeled as trend stationary with a one-time structural change (Kaufmann et al., 2010).

Finally, the ADF tests on the regression residual from Eq. (2), reported in the first column of Table 2, generally indicate that the stochastic trend in the time series for radiative forcing also is present in the time series for global surface temperature that are simulated by the climate models. For all experiments, save one (GFDL_1), the ADF statistic rejects the null hypothesis that the residual contains a stochastic trend. Moreover, the autocorrelations of the residuals (ρ_1 and ρ_2 in Table 2) generally indicate short-lived persistence. This implies that the stochastic trend in the exogenous forcing series that is used to run the 20c3m experiments appears in the time series for global surface temperature that is simulated endogenously by the AOGCM. This is consistent with the finding of cointegration between radiative forcing and temperature in the instrumental temperature record (Kaufmann and Stern, 2002; Kaufmann *et* al. 2006a) as well as surface temperature simulated by the one percent experiment, which is run for the CMIP2 (Kaufmann et al., 2006b).

The Perron (1997) tests (Table 2), seem to contradict the previous results in Tables 1 and 2, which are consistent with the stochastic trend/cointegration view: in all but three cases, the Perron test rejects the null hypothesis of a stochastic trend, against the alternative of the experimental temperature series being stationary around a linear trend with a one-time structural break. Somehow, the contradictory results from the Perron test, which favor the stationary trend-break model, need to be reconciled with all the other results which favor the stochastic trend/cointegration model.

Table 1 Unit root tests on radiative forcing

	ADF	Perron	L&P
Greenhouse gases	−0.34 (5)	−3.64 [1994] (4)	−5.10 [1866 1939] (5)
Sulfur emissions (SOx)	−1.96 (3)	−2.24 [1922] (4)	−7.08* [1947 1967] (3)
Solar insolation	−2.58 (5)	−3.51 [1888] (5)	−8.31** [1903 1944] (5)
Volcano	−4.78** (2)	−5.05* [1941] (2)	−6.93* [1883 1903] (2)
Anthropogenic & solar	2.06 (5)	−3.18 [1998] (1)	−5.90 [1898 1972] (5)

The column "ADF" reports the standard Augmented Dickey-Fuller t-statistic (Dickey and Fuller (1979) and Said and Dickey (1984)) on the restriction α=1 in the following regression: $\Delta y_t = \mu + \beta t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \varepsilon_t$ where y_t is the variable being tested, Δ is the first difference operator ($\Delta y_t = y_t - y_{t-1}$) and ε_t is a white noise error. The lag length k (given in parenthesis) is chosen by the Akaike information criterion. The column "Perron" reports the Perron (1997) test for a unit root allowing under the alternative for a break in the trend function at once. The test is conducted in two steps. Letting TB denote the date at which the break in the trend function occurs the series y_t being tested is first detrended using the regression $y_t = \mu + \beta t + \gamma DT_t + \widetilde{y}_t$ where $DT_t = 1(t > TB)(t - TB)$ and $\widetilde{y}$ is the regression residual. The test is then performed in the second step by using the t-statistic for α=1 in the regression $\widetilde{y}_t = \alpha\widetilde{y}_{t-1} + \sum_{i=1}^{k} c_i \Delta\widetilde{y}_{t-i} + e_t$ (Eq. (3b) of Perron (1997)). The break date TB (reported in square brackets) is chosen by the first method given in section 2.1 of Perron (1997) and the lag length k is chosen by the procedure described in section 2.2 of Perron (1997). The results do not change qualitatively when the other methods described in Perron (1997) are applied to select TB and k. The column "L&P" reports the Lumsdaine and Papell (1997) test for a unit root against an alternative hypothesis that the series is trend stationary around a linear trend with two break points. The test is based on the model $\Delta y_t = \mu + \beta t + \theta DU1_t + \gamma DT1_t + \omega DU2_t + \psi DT2_t + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \varepsilon_t$ where $DU1_t$ and $DU2_t$ are indicator variables for a mean shift occurring at times $TB1$ and $TB2$, respectively, $DT1_t$ and $DT2_t$ are corresponding trend slope shift variables. The test statistic for the null hypothesis α=0 is obtained by the sequential procedures described in detail in Banerjee et al. (1992) (see Lumsdaine and Papell (1997)). The two break points (reported in square brackets) and the lag length k (in parenthesis) are determined endogenously by the test procedure (see Banerjee et al. (1992) and Lumsdaine and Papell (1997)). In each test, * and **, respectively, indicates that the unit root null hypothesis is rejected at 5 % and 1 % level. The sample period is 1850–2000

4 Discussion

Results in the previous section indicate that;

- The time series for radiative forcing that are used to simulate AOGCMs contain a stochastic trend.
- This stochastic trend drives the non-stationary movements in global surface temperature that are simulated by the climate models.

These finding begs three important questions: (1) what are the physical mechanisms that impart a stochastic trend to radiative forcing, (2) how is the stochastic trend in radiative forcing imparted to global surface temperature, and (3) why do time series for observed and simulated temperature appear to be trend stationary with a one time structural change? In summary, the stochastic trends in radiative forcing are generated by the economic processes that generate anthropogenic emissions, and the physical and chemical processes that determine the atmospheric life times of radiatively active gases. These stochastic trends become

Table 2 Tests for cointegration between radiative forcing and global surface temperature simulated by 203 cm experiments

Experiment	ADF	ρ_1	ρ_2	Perron	L&P
BCCR	-4.49^{**} (2)	0.60	0.27	-5.74^{**}[1975] (3)	−5.96 [1895 1969] (2)
ECHAM5_1	-9.38^{**} (2)	0.34	−0.15	-10.21^{**}[1967] (1)	−7.72 [1933 1973] (3)
ECHAM5_2	-3.93^{**} (5)	0.27	−0.30	−4.25 [1976] (5)	−5.76 [1916 1963] (5)
ECHAM5_3	-6.05^{**} (2)	0.34	−0.15	-10.95^{**} [1975] (1)	-7.08^{*} [1939 1972] (4)
ECHAM5_4	-5.82^{**} (3)	0.34	−0.19	-6.50^{**} [1962] (3)	−6.08 [1889 1954] (5)
GFDL_1	−2.42 (3)	0.75	0.59	-5.42^{**} [1888] (0)	−4.60 [1883 1974] (3)
GFDL_2	-3.64^{*} (2)	0.56	0.31	−4.15 [1979] (5)	−5.44 [1893 1963] (2)
GFDL_3	-3.38^{*} (2)	0.53	0.38	−3.55 [1972] (2)	−5.34 [1882 1962] (2)
HADLEY_1	-4.05^{**} (2)	0.47	0.13	-5.93^{**} [1962] (2)	−6.47 [1917 1945] (2)
HADLEY_2	-4.78^{**} (2)	0.45	0.13	-7.86^{**} [1952] (0)	-6.67^{+} [1900 1948] (2)
GISS_1	-5.80^{**} (1)	0.52	0.34	-5.78^{**} [1964] (1)	−5.10 [1893 1960] (5)
GISS_2	-5.60^{**} (1)	0.52	0.29	-6.38^{**} [1973] (0)	-7.45^{**} [1925 1961] (0)
IPSL	-8.06^{**} (1)	0.22	−0.05	-9.75^{**} [1968] (0)	−6.21 [1897 1967] (4)

The column "ADF" reports the ADF t-statistic on the residual $\widehat{\mu}_t$ from an OLS regression of *Temp* on *F* (see Eq. 2). The ADF test is as in Table 1 except that the test regression assumes no time trend (β=0). The column "Perron" and "L&P", respectively, reports corresponding t-statistics based on the Perron (1997) test and the Lumsdaine and Papell (1997) test. The lag length and the endogenously determined break dates are obtained and reported as in Table 1. In each test, $^{+}$, *, and ** indicates that the unit root null hypothesis is rejected at the 10 %, 5 %, and 1 % level, respectively. Finally, ρ_j, j=1,2 indicates the OLS estimate of the autoregressive coefficient from $\widehat{\mu}_t = \rho_j \widehat{\mu}_{t-j} + e_t$. The sample period is 1850–2000

embodied in the global surface temperature via the physical relationship between radiative forcing and temperature. We illustrate this mechanism by showing that cointegration is implied mathematically by feeding stochastically trending forcing variables into a zero-dimensional energy balance model. In contrast, these relations cannot be explained as trend stationary process with a one-time break. Rather, the apparent stationarity of the model outputs around a broken trend seems to be a statistical artifact from the addition of internal weather noise to the temperature effects imparted by the stochastic trend in radiative forcing.

4.1 Economic and physical mechanisms that impart a stochastic trend to radiative forcing

One set of mechanisms that imparts a stochastic trend to the radiative forcing of radiatively active gases are the economic processes that generate anthropogenic emissions. For example, anthropogenic emissions of carbon dioxide and sulfur dioxide originate largely from the combustion of fossil fuels, which is driven in part by and cointegrate with, economic activity (e.g. Dinda and Coondoo, 2006). Methane emissions are associated with energy production and agricultural activities (Stern and Kaufmann, 1996). Finally, CFC emissions are largely associated with the production (and leakage) of refrigeration and air-conditioning equipment (Molina and Rowland, 1974).

The role of economic activity in anthropogenic emissions is important because economists routinely model economic variables as having a stochastic trend as opposed to being trend stationary with or without a structural change (Nelson and Plosser, 1982; West, 1988; King et al. 1991). The importance of representing emissions as having a stochastic trend as opposed to a deterministic trend is highlighted by two important changes in emissions. The collapse of the former Soviet Union causes annual carbon emissions by Centrally Planned

Europe and Eurasia to decline from 1.41 billion tons in 1988 to 0.80 billion tons in 1999 (Marland et al., 2008). Since 1999, emissions resume growing. These changes are better modeled as part of a stochastic trend in Soviet/post-Soviet economic activity than a trend stationary process with several structural changes in the slopes and/or intercepts.

Equally significant is the slow-down and reversal in anthropogenic sulfur emissions. This change is associated with the legislation in North America, Western Europe, and Japan that aims to slow acid deposition. Following one method of compliance, firms install scrubbers, which reduce the amount of sulfur reaching the atmosphere. Their installation represents a long lasting and ongoing change in capital stock, not a one-time change in the slope or intercept of a trend stationary process. Consistent with these arguments and previous results, ADF statistics indicate that emissions of carbon dioxide[2] (−0.85) and sulfur (−1.57) contain a stochastic trend (as opposed to being stationary) and the test statistic developed by Perron (1997) fails to reject ($p>0.50$) the null hypothesis that anthropogenic emissions of carbon dioxide (−3.15) or sulfur (−2.01) contain a stochastic trend against the alternative that the time series for emissions are trend stationary with a one-time structural change.

A second mechanism that imparts a stochastic trend to radiative forcing is the tendency of the atmosphere to integrate anthropogenic emissions of some gases. The atmospheric concentration of greenhouse gases can be modeled using a version of Eq. (1):

$$C_t = \theta C_{t-1} + e_t \tag{5}$$

in which C is the atmospheric concentration of a gas in year t, e are anthropogenic emissions of that gas, and θ is the autoregressive coefficient. If θ has a value that is equal to or close to one, the atmosphere will integrate anthropogenic emissions to create a stochastic trend in atmospheric concentrations. Because radiative forcing is closely correlated with concentrations, a stochastic trend in concentrations will create a stochastic trend in radiative forcing.

The current understanding of atmospheric chemistry implies that several greenhouse gases have θ values that are nearly one, as indicated by their atmospheric lifetimes. The lifetime of a gas in the atmosphere is defined by the period it takes for a perturbation to be reduced to 37 % of its initial amount (Meehl et al. 2007). The lifetime for many of the important greenhouse gases is measured in decades or centuries. For example, nitrous oxide has a lifetime of 114 years (Forster et al., 2007). CFC11 and CFC12 have lifetimes of 45 (Forster et al., 2007) and 87 years respectively (Volk et al., 1997). These lifetimes imply values for θ of 0.991 (nitrous oxide), 0.978 (CFC11) and 0.989 (CFC12).[3] These values will impart a stochastic trend to the radiative forcing of these gases. Unlike these gases, it is not possible to calculate a lifetime for CO_2 (Meehl et al. 2007). Nonetheless, models indicate that a significant fraction of the carbon emitted by the combustion of fossil fuels remains in the atmosphere for centuries (Archer et al., 2009).

These two sources of stochastic trends are consistent with statistical results regarding the time series properties of forcings that are affected by human activity. For example, a suite of statistical tests indicate that the radiative forcing of CFC's are integrated order two (Stern and Kaufmann, 2000). Integrated order two I(2) implies that the term η in Eq. 1 contains a stochastic trend, and the autocorrelation coefficient $\theta=1$ integrates the stochastic trend in η a second time to create Y.[4] As such, the I(2) results for the radiative forcing of CFC's may be

[2] CO2 emissions are the sum of emissions (1850–2000) from land-use change (Houghton, 2008) and emissions from fossil fuels and cement production (Marland et al. 2008).

[3] For example, for CFC12 we have $0.37=\theta^{87}$, or $\theta=0.37^{1/87}\approx 0.989$.

[4] A stochastic process is called integrated of order d, $I(d)$, d=0,1,2, …, if $\Delta^d X_t$ is $I(0)$, where $I(0)$ is a stationary process and Δ is the difference operator, $\Delta X_t = X_t - X_{t-1}$.

generated by the atmosphere integrating the stochastic trend in anthropogenic emissions, which is generated by emissions from the stock of refrigeration and air conditioning equipment. Similar mechanisms may generate results that suggest the atmospheric concentration (and radiative forcing) of CO_2 and nitrous oxide is integrated order two. Conversely, the results for methane and SOx clearly are integrated order one. For both of these gases, the short atmospheric residence times (a decade for methane, less than two weeks for SOx) will not integrate the stochastic trend in anthropogenic emissions that is imparted by energy using capital stock. When individual forcings are summed together, higher orders of integration associated with CO_2 and CFC should dominate asymptotically, which would make global surface temperature integrated order two. However, formal statistical tests indicate that the aggregate forcing series is integrated of order one rather than two.

In contrast to these explanations for the stochastic trends in emissions and atmospheric concentrations of radiatively active gases, very special conditions are required to generate concentrations that are trend stationary with a one-time structural change. Using Eq. (4) to represent concentrations implies that they increase at a constant rate γ (plus a random year-to-year variation). To generate a constant increase in atmospheric concentrations γ, (or $\gamma+\lambda$ after T_b) Eq. (4) implies that anthropogenic emissions (e_t) must increase each year by $\gamma t-\theta C_t$ plus stationary noise. This balance seems unlikely.[5] And without this balance, concentrations will not increase at the constant rate that is implied by γ in Eq. (4).

4.2 Imparting the stochastic trend in radiative forcing to global surface temperature

The cointegration/error correction models specify the transmission of the stochastic trends in radiative forcing to global surface temperature in a way that is consistent with the basic physics of the climate system. These physical relationships can be approximated with a simple zero-dimensional energy balance model:

$$C\frac{dTemp}{dt} = F - A - BTemp + w_t \tag{6}$$

in which *Temp* is surface temperature (70 m mixed layer of the ocean), C is this layer's effective heat capacity, F is the external radiative forcing (both short and long wave), the term A-B*Temp* is a linearization of outgoing long wave radiation, and w is a stochastic heat flux forcing of the ocean surface associated with atmospheric 'weather noise" (e.g. Mann, 2011). Solving this equation for d*Temp*/dt

$$\frac{dTemp}{dt} = a_1 F - k_1 - b_1 Temp + \widetilde{\varepsilon}_t \tag{7}$$

in which a_1 is $\frac{1}{C}$, k_1 is $\frac{A}{C}$, b_1 is $\frac{B}{C}$, and $\widetilde{\varepsilon}_t$ is w_t/C. Eq. (7) is the continuous-time counterpart of the discrete-time cointegration/error correction model that is generated by substituting Eq. (2) into Eq. (3):

$$\Delta Temp_t = a_2 F_{t-1} - k_2 - b_2 Temp_{t-1} + \varepsilon_t \tag{8}$$

in which a_2 is $-\rho\beta$, k_2 is $(-(\alpha_2 + \rho\alpha_1))$, and b_2 is $-\rho$. More precisely, (8) is the Euler discrete-time approximation to the stochastic differential Eq. (7). Equation (8) does not

[5] This balance is even more complex for radiative forcing of many gases, which are nonlinearly related to concentrations (Eqs. 6–8).

include the lagged first differences of the temperature and radiative forcing from the error correction model because statistical estimates indicate that the coefficients associated with these variables (ϕ*'s* and ψ*'s* Eq. 3) are not statistically different from zero (e.g. Kaufmann et al. 2006a).

Equation (8) implies that, if forcing has a stochastic trend, then temperature and forcing are cointegrated; but (8) is the discrete-time version of (7), so this same conclusion follows from the continuous-time energy balance model (6). As such, our interpretation of Eqs. (6) and (8) is consistent with the finding that model inputs (radiative forcing) and output (temperature) cointegrate. The equilibrium condition $a_2F - k_2 - b_2Temp = 0$ defines the (long-run) temperature sensitivity $\Delta Temp_{2x}$. Because of this consistency, the cointegration/error correction model is used to estimate the transient climate response from empirical data of 2.1 °C (Kaufmann et al., 2006a) and can accurately recover the transient climate response from temperature data generated by AOGCMs (Kaufmann et al., 2006b).

Furthermore, the correspondence between the energy balance model (Eq. 6) and the cointegration/error correction model (Eq. 8) allows us to compare the e-folding rate (τ) with our estimate for the rate of error correction (ρ). By definition, $\tau = C/B$ which is equals the inverse of b_2 ($b_2=-\rho$) in Eq. (8). The values for C and B used by Mann (2011) imply $\rho = \frac{1}{5.3} = -0.19$. Point estimates of ρ (Eq. 3) are −0.24, −0.39, −0.82 from the 1864–1998, 1920–1998, and 1960–1998 sample periods, respectively (Kaufmann et al., 2011).

Finally, the error correction term allows the statistical analysis to go beyond the finding of a simple correlation between temperature and radiative forcing. As reported by Kaufmann et al.(2006a; 2011), statistical estimates for the value of ρ (Eq. 3) indicate that temperature adjusts to disequilibrium (μ) in the cointegrating relation between temperature and radiative forcing. But if Eqs. (2) and (3) are specified such that radiative forcing (F) is the dependent variable and temperature (Temp) is the independent variable, estimates for ρ in Eq. (3) are not statistically different from zero, which indicates that radiative forcing does not adjust to temperature. This is consistent with the understanding of climate given by Eq. (6).

Conversely, models derived from the trend stationary approach cannot be interpreted relative to Eq. (6). Climate sensitivity cannot be calculated from statistical estimates of Eq. (4). Moreover, removing a deterministic trend from the time series for temperature and radiative forcing weakens the ability to allocate changes in temperature to radiative forcing.

4.3 Why do time series for observed and simulated temperature appear to be trend stationary with a one time structural change?

The inability of trend stationary models to represent the accumulation of anthropogenic emissions into concentrations, and the relation between radiative forcing and temperature begs the question, why are results generated by statistical tests consistent with the hypothesis that global surface temperature is trend stationary with a one time structural change? We propose that weather noise adds internal variability to the stochastic trend imparted by the time series for radiative forcing (F) and this ‘tricks’ the test statistic developed by Perron (1997) such that it rejects the null hypothesis that the temperature time series contains a stochastic trend.

To evaluate the effect of weather noise on results generated by the Perron (1997) test statistic, we create one thousand experimental data sets, each of which is the aggregate for the radiative forcing of greenhouse gases, sulfur emissions, and solar insolation plus white noise (normally distributed with mean zero and standard deviation of 0.2). This white noise is akin to internal climate variability (w in Eq. 6) added by the climate model. As indicated in Table 2, the autocorrelation coefficients on the regression residual from Eq. 2 are relatively small and the 0.2 value is chosen to mimic the approximate size of the observed climate variability (e.g. Crowley, 2000). Each of the experimental time series is analyzed with the test statistic developed by Perron (1997). The test statistic rejects the null hypothesis ($p=$ 0.05) for 908 of the 1,000 data sets. Results that reject the null hypothesis for more than 5 percent of the experimental time series do not depend on the size of the standard deviation of the white noise—using a value of 0.05 (instead of 0.2) to create the experimental data sets generates results in which the test statistic rejects the null hypothesis for 437 of the 1,000 data sets. The high rate of rejection (fifty rejections are expected based on random chance) incorrectly gives the impression that that the simulated experimental time series for radiative forcing (plus white noise) is trend stationary with a one-time break. Furthermore, the mean break date, 1965±6.63 is similar to those reported from analyses of the instrumental temperature record (Gay et al., 2009) or the time series for global surface temperature (Estrada et al., 2011).

Asymptotically, adding white noise to a time series should not alter conclusions about its time series properties, but in small samples, a large white noise component makes it harder to distinguish the inherent I(1) property (Stock, 1994). Figure 2 illustrates the point. If there is no noise, the observed values of radiative forcing (red line) rarely cross the lines generated by the trend stationary model with a one-time break (blue line). Adding white noise (black line) allows the simulated experimental time series to cross the time series simulated by the TSOTSC model several times in both segments. These crossings make the experimental time series incorrectly appear stationary around a deterministic trend.

This interpretation is strengthened by results in Table 2, which indicate that the simulated temperature time series cannot be modeled as trend stationary if more than one break is

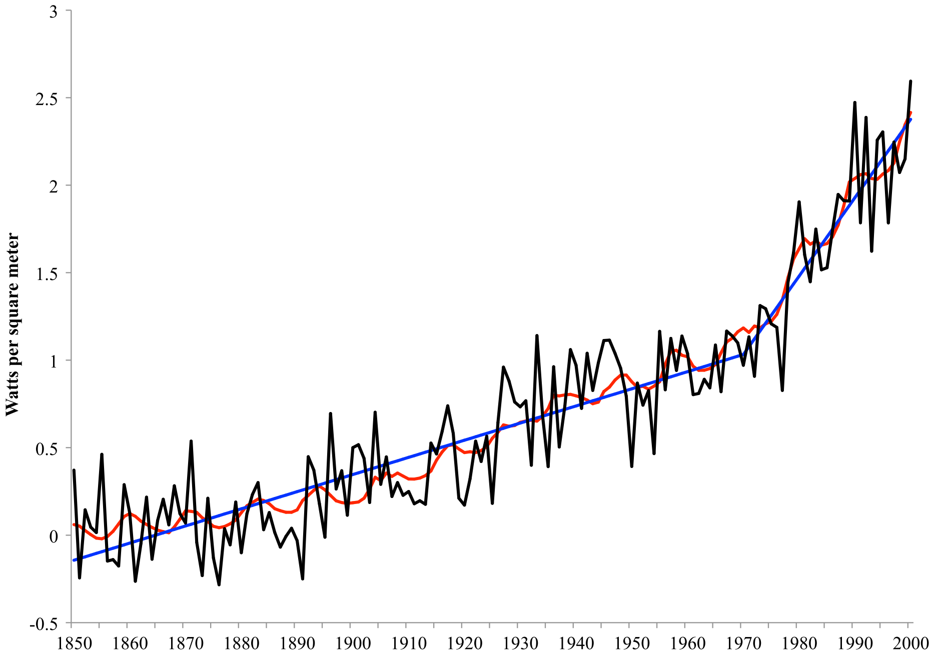

Fig. 2 The aggregate for radiative forcing (*red line*), the aggregate for radiative forcing plus white noise (*black line*), and the trend stationary with a one-time break (*blue line*). The break date for the blue line is 1973 and the Perron (1997) test statistic is −7.19, which rejects the null hypothesis at the five percent significance level

allowed. The Lumsdaine and Papell (1997) test statistic rejects the unit root for only three of the thirteen temperature simulations analyzed. The smaller number of rejections relative to the ten rejections indicated by the Perron (1997) statistic is consistent with an analysis of economic time series reported by Lumsdaine and Papell (1997).

5 Conclusion

Statistical tests suggest that it is possible to model the time series for global surface temperature as either trend stationary (with or without a structural change) or containing a stochastic trend. But it is difficult to describe a physical mechanism that generates a time series for global surface temperature following a deterministic trend. The data generating process for the time series of anthropogenic emissions or the radiative forcings that are used to simulate the 20c3m models cannot be modeled as TSOTSC. Nor can a trend stationary process be used to describe a physical mechanism for changes in radiative forcing or temperature per se. Instead, the finding that temperature is trend stationary with a break appears to be associated with adding internal climate variability to the relatively “smooth” stochastic trend in radiative forcing.

Conversely, the statistical results are consistent with the hypothesis that the time series for radiative forcing contain a stochastic trend and that this trend is imparted to global surface temperature. The stochastic trends in radiative forcing are generated by the economic forces that drive anthropogenic emissions and the atmospheric chemistry that determines the rate at which emissions accumulate in the atmosphere. Finally, a zero dimensional energy balance model is consistent with the notion that a stochastic trend in radiative forcing is communicated directly to temperature. Together, these results indicate that using the ideas of stochastic trends, cointegration, and error correction (as opposed to TSOTSC models) can generate reliable conclusions regarding the causes of changes in global surface temperature during the instrumental temperature record and more reliable forecasts of how temperature will evolve in the future.

References

Archer DM, Eby V, Brovkin A, Ridgwell L, Cao U, Mikolajewicz K, Caldeira K, Matsumoto G, Munhoven AM, Tokos K (2009) Atmospheric lifetime of fossil fuel carbon dioxide. Annu Rev Earth Planet Sci 37:117–134. doi:10.1146/annurev.earth.031208.100206

Banerjee A, Lumsdaine RL, Stock JH (1992) Recursive and sequential tests of the unit-root and trend-break hypotheses: theory and international evidence. J Bus Econ Stat 10(3):271–288

Bloomfield P, Nychka D (1992) Climate spectra and detecting climate change. Climate Change 21:275–287

Crowley TJ (2000) Causes of climate change over the past 1000 years. Science 289:270–277

Dickey DA, Fuller WA (1979) Distribution of the estimators for autoregressive time series with a unit root. J Am Stat Assoc 74:427–431

Dinda S, Coondoo D (2006) Income and emissions: panel data-based cointegration analysis. Ecol Econ 57 (2):167–181

Estrada F, Gay C, Sanchez A (2010) Reply to “Does temperature contain a stochastic trend? Evaluating conflicting results by Kaufmann et al. Clim Chang 101(3–4):407–414. doi:10.1007//s10584-010-9928-0

Estrada F, Perron P, Gay-Garcia C, Martinez-Lopez B (2011), A time-series analysis of the 20th century climate simulations produced for the IPCC’s AR4, http://ideas.repec.org/p/bos/wpaper/wp2011-051.html

Forster P, Ramaswamy V, Artaxo P, Berntsen T, Betts R, Fahey DW, Haywood J, Lean J, Lowe DC, Myhre G, Nganga J, Prinn R, Raga G, Schulz M, Van Dorland R (2007) Changes in atmospheric constituents and in radiative forcing. In: Solomon S, Qin D, Manning M, Chen Z, Marquis M, Averyt KB, Tignor M, Miller

HL (eds) Climate change 2007: The physical science basis. Contribution of working group I to the fourth assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge

Gay C, Estrada F, Sanchez A (2009) Global and hemispheric temperature revisited. Clim Chang 94:333–349. doi:10.1007/s10584-008-9524-8

Hansen JE, Sato M (2001) Trends of measured climate forcing agents. Proc Natl Acad Sci 98:14778–14783. doi:10.1073/pnas.261553698

Harvey LD, Kaufmann RK (2002) Simultaneously constraining climate sensitivity and aerosol radiative forcing. J Clim 15(20):2837–2861

Houghton RA (2008) Carbon flux to the atmosphere from land-use changes: 1850–2005. In TRENDS: A compendium of data on global change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A

Kaufmann RK, Kauppi H, Stock JH (2006a) Emission, concentrations, & temperature: a time series analysis. Clim Chang 77:249–278

Kaufmann RK, Kauppi H, Stock JH (2006b) The relationship between radiative forcing and temperature: what do statistical analyses of the instrumental temperature record measure? Clim Chang 77:279–289

Kaufmann RK, Kauppi H, Stock JH (2010) Does temperature contain a stochastic trend? Evaluating conflicting statistical results. Clim Chang 101:395–405

Kaufmann RK, Kauppi H, Mann ML, Stock JH (2011) Reconciling anthropogenic climate change with observed temperature 1998–2008. Proc Natl Acad Sci 108(29):11790–11793. doi:10.073/pnas.1102467108

Kaufmann RK, Stern DI (2002) Cointegration analysis of hemispheric temperature relations. J Geophys Res. 107 D2 10.1029, 2000JD000174

King RG, Plosser CI, Stock JH, Watson MW (1991) Stochastic trends and economic fluctuations. Am Econ Rev 81(4):819–840

Lean J (2004). Solar irradiance reconstruction. IGBP PAGES/World Data Center for Paleoclimatology Data Contribution Series # 2004–035. NOAA/NGDC Paleoclimatology Program, Boulder CO, USA

Lindzen RS, Giannitsis C (1998) On the climatic implications of volcanic cooling. J Geophys Res 103:5929–5941

Lumsdaine RL and Papell DH (1997) Multiple trend breaks and the unit-root hypothesis, The Review of Economics and Statistics, pp 212–218

Mann ME (2011) On long range temperature dependence in global surface temperature series. Clim Chang 107:267–276

Marland G, Boden TA, Andres RJ (2008) Global, regional, and national fossil fuel CO2 emissions. In Trends: A compendium of data on global change. Carbon Dioxide Information Analysis Center, Oak Ridge National Laboratory, U.S. Department of Energy, Oak Ridge, Tenn., U.S.A.

Meehl GA, Stocker TF, Collins WD, Friedlingstein P, Gaye AT, Gregory JM, Kitoh A, Knutti R, Murphy JM, Noda A, Raper SCB, Watterson IG, Weaver AJ, Zhao Z-C (2007) Global climate projections. In: Solomon S, Qin D, Manning M, Chen Z, Marquis M, Averyt KB, Tignor M, Miller HL (eds) Climate change 2007: The physical science basis. Contribution of working group I to the fourth assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge

Mills TC (2009) How robust is the long-run relationship between temperature and radiative forcing? Clim Chang 94:351–361. doi:10.1007/S10584-008-9525-7

Molina MJ, Rowland FS (1974) Stratospheric sink for chloroflouromethanes: chlorine atom-catalyzed destruction of ozone. Nature 249:810–812. doi:10.1038/249810a0

Nelson CR, Plosser CI (1982) Trends and random walks in macroeconomic time series: some evidence and implications. J Monetary Econ 10:139–162

Perron P (1997) Further evidence on breaking trend functions in macroeconomic variables. J Econometrics 80 (2):355–385

Rea W, Reale M, Brown J (2011) Long memory in temperature reconstructions. Clim Chang 107:247–265. doi:10.1007/s10584-011-0068-y

Said E, Dickey DA (1984) Testing for unit roots in autoregressive moving average models of unknown order. Biometrika 71:599–607

Sato M, Hansen JE, McCormick MP, Pollack JB (1993) Stratospheric aerosol optical depths, 1850–1990. J Geophys Res 98:22987–22994

Stern DI, Kaufmann RK (1996) Estimates of global anthropogenic methane emissions 1860–1993. Chemosphere 33(1):159–176

Stern DI, Kaufmann RK (2000) Is there a global warming signal in hemispheric temperature series: a structural time series approach. Clim Chang 47:411–438

Stock JH (1994) Unit roots, structural breaks, and trends, ch. 46. In: Engle R, McFadden D (eds) Handbook of econometrics, volume IV. Elsevier, Amsterdam, pp 2740–2843

Volk CM, Elkins JW, Fahey DW, Dutton GS, Gilligan JM, Loewenstein M, Podolske JR, Chan KR, Gunson MR (1997) Evaluation of source gas lifetimes from stratospheric observations. J Geophys Res-Atmos 102 (D21):25543–25564

West KD (1988) Asymptotic normality when regressors have a unit root. Econometrica 56:1397–1418

Woodward WA, Gray HL (1993) Global warming and the problem of testing for trend in time series data. J Clim 6:953–962

Woodward WA, Gray HL (1995) Selecting a model for detecting the presence of a trend. J Clim 8:1929–1937